TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 681

REDUCTION OF WING LIFT BY THE DRAG

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REDUCTION OF WING LIFT BY THE DRAG*

By A. Betz and J. Lotz

By assuming pure potential flow, the lift of an airplane wing can be theoretically computed. In this the empiricism is used that the trailing edge of the profile is not in the circulation but rather that the fluid particles from the pressure and the suction side meet precisely at the trailing edge. There is no drag in this theoretical flow, because a body experiences no drag in a nonvortical potential flow. Now, in fact, each flow around a body deviates to a certain extent from a potential flow. There is always a more or less extended wake (dead air) aft of the body, because of the friction of the fluid on the surface. The result is the appearance of a drag and, in the airplane wing, in particular, a lower lift as compared to the theoretical figure. Since both actions are traceable to the same cause, it indicates some connection between the amount of drag and that of the reduced lift.

The problem probably yields to theoretical treatment on the basis of the following: The streamlines on the wing surface are displaced by the retarded flow on the surface (friction layer). Such a flow can be approximately reproduced as a potential flow by disposing sources on the wing surface which effect the identical displacement of the streamlines. (Fig. 1.) Then the dead-air space is so filled up by the flow from the sources that a potential flow is produced, whose streamlines approximately correspond with those of the actual flow. The dead air space on the wing in normal attitude forms primarily on the suction side, so these sources are ordinarily arranged on the suction side. Now the conformal transformation of the wing to a circle (fig. 2) makes the produced flow readily amenable to theoretical analysis. Trailing edge H becomes H_1 of the periphery. For a pure potential flow the flow merges precisely at trailing edge H, thus H_1

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*"Verminderung des Auftriebes von Tragflügeln durch den Widerstand." Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 28, 1932, pp. 277-279.

must be the stagnation point in the flow round the circle. The magnitude of the circulation is defined by this stipulation. Now the disposal of sources* on the suction side, respectively, its corresponding part on the periphery, causes, because of their one-sided position, an additional velocity in point H_1, which displaces the stagnation point, say, to H_1'.

Now, the circulation, and because of it, the lift, must become lower, so that the stagnation point is shifted again to point H_1 corresponding to the trailing edge. From this demand the requisite reduction in circulation and lift can be calculated. On the other hand, the calculation of the drag from the velocity distribution in the dead air space, which corresponds to the source distribution, is also known,** so that the relationship between reduced lift and drag can be defined. The real obstacle to such a calculation is primarily the somewhat conclusive knowledge of the boundary layer necessary to assure a correct source distribution. Owing to this drawback, the problem has never been attacked quantitatively heretofore, but is in process of preparation at the present time.

An experimental answer to this question of relation between reduced lift and drag was first attempted by C. Wieselsberger.*** He utilized the measurements of a wing whose drag had been systematically altered by increasing the roughness on the suction side. By plotting the polars (c_a against c_w) extrapolated for infinite aspect ratio (span : wing chord) for the different degrees of roughness of the wings, and adding the theoretical values, the points of equal angles of attack are, according to Wieselsberger, approximately on parallel straight lines. This means that the decrease in lift figure Δc_a is proportional to the drag figure c_w. And Wieselsberger's method reveals that

$$\Delta c_a \approx 7.5 c_w \tag{1}$$

*To compute the flow around the circle, these sources as well as the corresponding sink must be reflected at infinity on the circle. The reflected sink lies in the center of the circle.

**A. Betz: Z.F.M., No. 16, 1925, p. 43.

***C. Wieselsberger: The most important results of the wing theory and its proof by experiment, in v. Karman-Levi-Civita Vorträge aus dem Gebiete der Hydro- und Aeromechanik (Innsbruck, 1922). J. Springer, Berlin, 1924.

Now it is obvious of itself that this simple relation cannot be generally valid, for in symmetrical wings at least Δc_a changes sign with the angle of attack (with the lift), whereas c_w is always positive. Added to this, Wieselsberger's study had been limited to one single airfoil section and one fully defined type of drag production, i.e., roughness of suction side. Consequently, there still remained a need to attack this question of interdependence between drag and reduced lift from a different angle also.

With this in view, several measurements on Joukowsky wing sections* are evaluated, in which the theoretical lift can be comparatively easily computed and compared with the actually measured lift and with the measured drag. Admittedly, the accuracy of this investigation is not very great, because ordinarily, the profile drags and the lift differences are quite small, so that the inaccuracies in measuring and in model construction appear relatively of as great consequence. Besides, the usual extrapolation for infinite aspect ratio is no longer wholly admissible for the wing with separated flow which was included in the study. But in spite of this, some practical results were anticipated. The profiles were tested at two speeds (15 and 30 m/s = 49.2 and 98.4 ft./sec.), so that of itself an effect of the Reynolds Number would have been ascertained. The analysis was likewise made for the two speeds. But, on the whole, the discrepancies were not pronounced enough to permit conclusions to be drawn therefrom. Hence the results, in the following, are restricted to those at 30 m/s.

The profiles used for the study are shown in Figure 3. The selection assured a very large range of forms. The Joukowsky sections have, as is known, two typical quantities: one, f/t, representing approximately the camber of the median line; the other, d/t, approximately the relative thickness of the profiles (fig. 4, $t \approx$ profile chord, $f \approx$ midordinate of center line, $d \approx$ profile chord in center.** These quantities have been appended in

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*Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Report III, p. 59. Compare also Z.F.M., Vol. 18, 1927, p. 225.

**The exact significance results from the design of these profiles. See, for example, Ergebnisse der Aerodynamischen Versuchsanstalt zu Gottingen, Report III, p. 15, where, however, $t = 2l$ and d thereofre defined as half as high as here.

Figure 3, and subsequently included in the data. In Figure 5, profile $f/t = 0.5$, $d/t = 0.25$ is plotted with its measured lift coefficients c_a against drag coefficients c_w for ∞ aspect ratio, the points with the corresponding theoretical c_a, c_w values (the theoretical $c_w = 0$) being shown connected by straight lines for a number of angles of attack. This method of representation conforms to Wieselsberger's method. It is seen that these straight lines are, in fact, somewhat parallel - at least, in a certain range. But it also becomes clearly apparent that the slope of the straight lines certainly is not the same in general.

To bring out the conditions more clearly, lift loss Δc_a is given against drag coefficient c_w in Figures 6 to 10 for various profiles, Δc_a being the difference between theoretical and actual lift coefficient. The profiles having equal mean camber are given in one plot. For comparison, the straight line which corresponds to Wieselsberger's relation $\Delta c_a = 7.5\ c_w$ has been included as dashed line. It is seen that this latter relation signifies, at any rate in a limited range, about at lifts above that at $c_{w\ min}$, a passable approximation, especially with the proviso to select instead of factor 7.5, one that is more suited to a specific profile form. But it is also apparent that, in general shape, the curves distinctly evince a pronounced conformity of altogether different type.

The course may be approximately described as follows: The curves for profiles of equal mean camber are essentially symmetric to one common center line, whose form is dependent on the profile camber only. (Fig. 11, dashed line.) In symmetrical profiles this center line is straight, elsewhere curved. The form of the superimposed curve depends primarily on the profile thickness. Thus, the relation between lift loss and drag can be approximately divided into two constituent parts: one, depending on the camber only, the other, on the thickness only.

$$\Delta c_a = F_1\ (c_w,\ f/t)\ \pm F_2\ (c_w,\ d/t) \qquad (2)$$

After all, the influence of the thickness is mostly not as clearly expressed as that of the camber. It is only in very thin profiles that a striking narrowing of the curve becomes eminent, which, by simultaneously strong camber, forms even an intersection. (Figs. 9 and 10.)

Owing to this increase in thickness effect with the camber the above cited division into an effect of camber and one of thickness is not altogether conclusive as far as the latter is concerned.

SUMMARY

Drag and loss of lift of a wing are attributable to the same cause, wake formation, thus indicating that there is some relation between both. The basic factors for theoretical treatment of this problem are available, but such studies have not been pursued heretofore. By experiment, Wieselsberger established a very simple relationship, although of limited range of validity. The analysis of measurements on Joukowsky sections revealed a typical course of the curves for the interdependence between drag and loss of lift. The shape of the curves apparently depends quite regularly on the mean camber and on the thickness of the profile.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

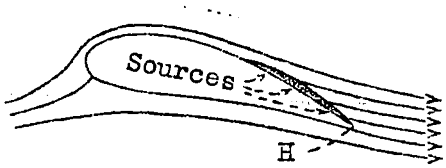

Fig. 1 Wing with sources on suction side

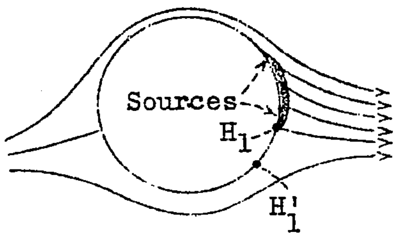

Fig. 2 Conformal transformation of wing with sources in a sphere

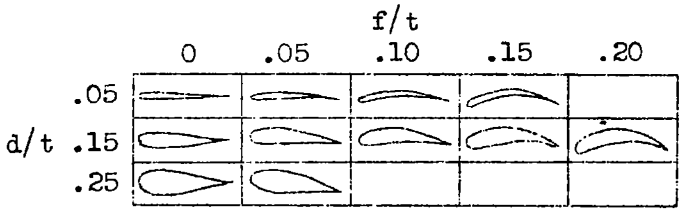

Fig. 3 Analyzed Joukowsky airfoils

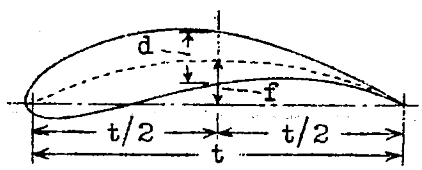

Fig. 4 Characteristic quantities f, d and t

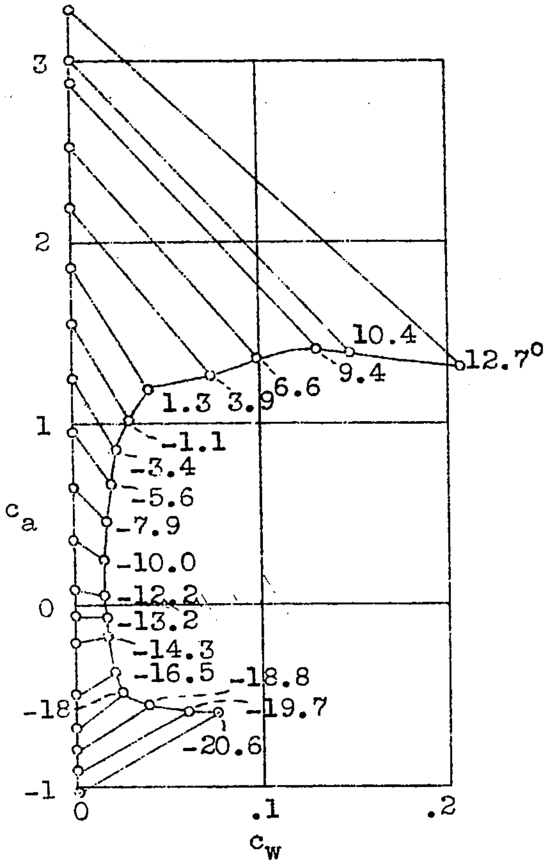

Fig. 5 c_a plotted against c_w for Joukowsky airfoil
$f/t = 0.05$, d/t 0.25

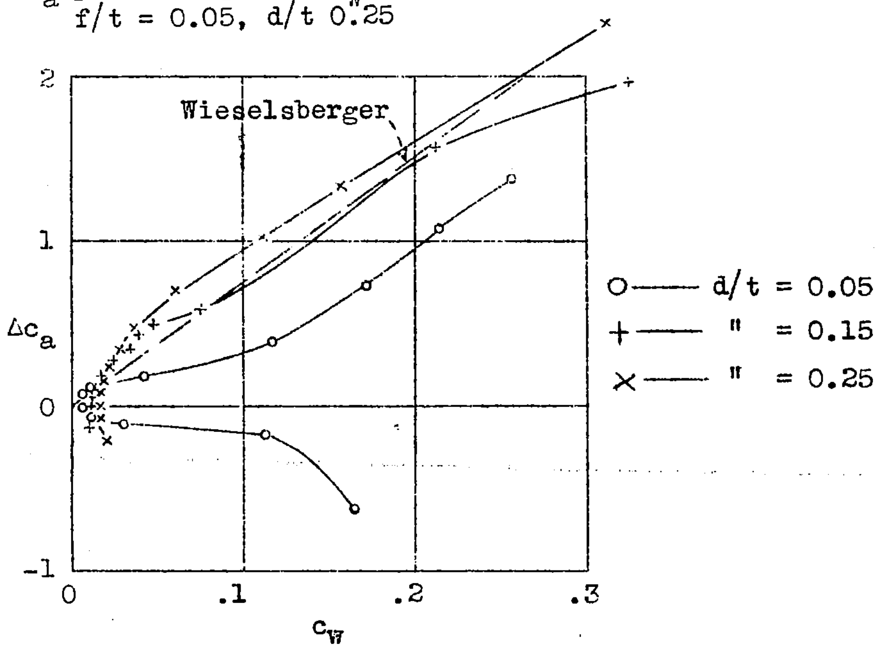

Fig. 6 Symmetrical wing sections ($f/t = 0$)

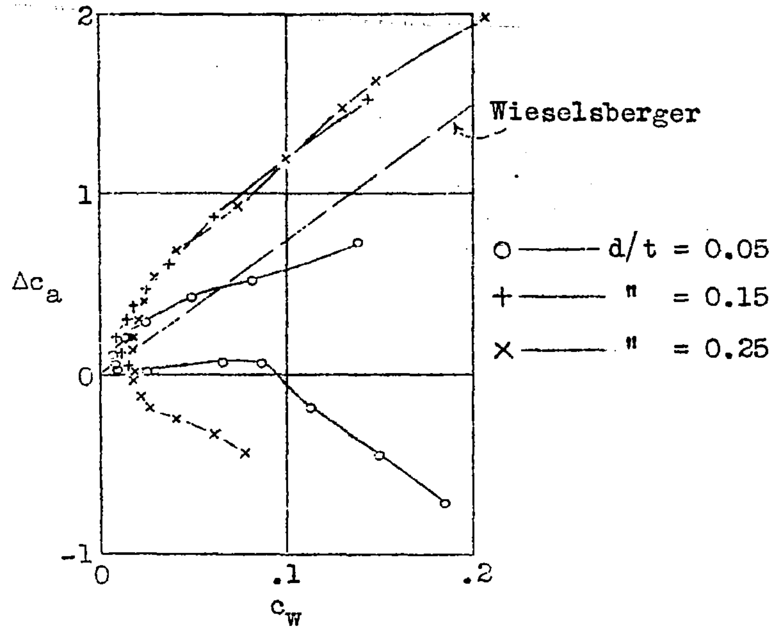

Fig. 7 Profils with f/t = 0.05

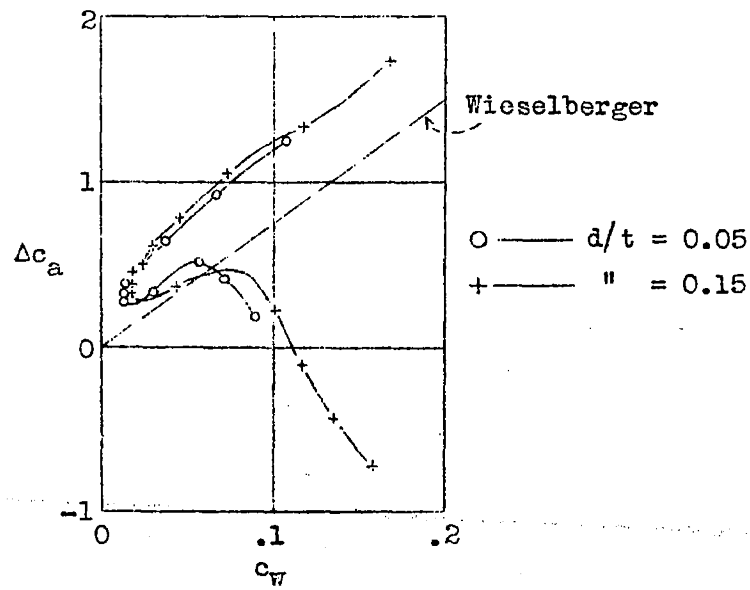

Fig. 8 Profils with f/t = 0.10

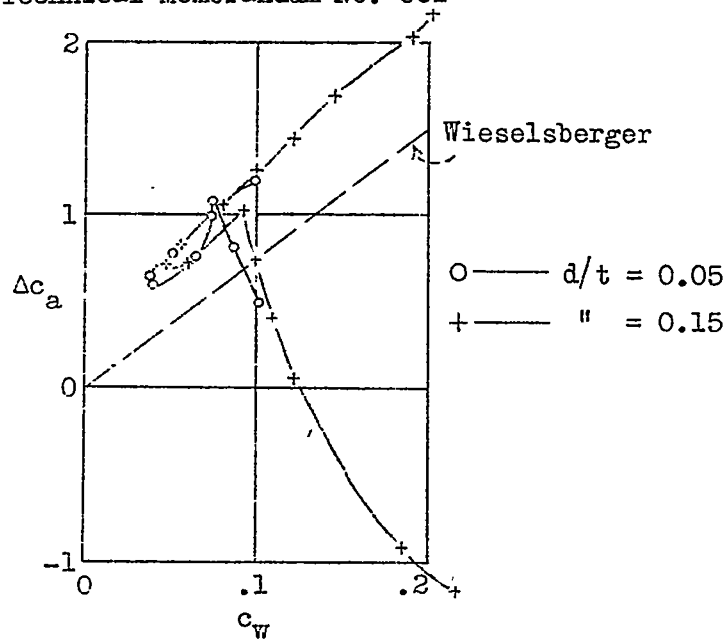

Fig. 9 Profile with f/t = 0.15

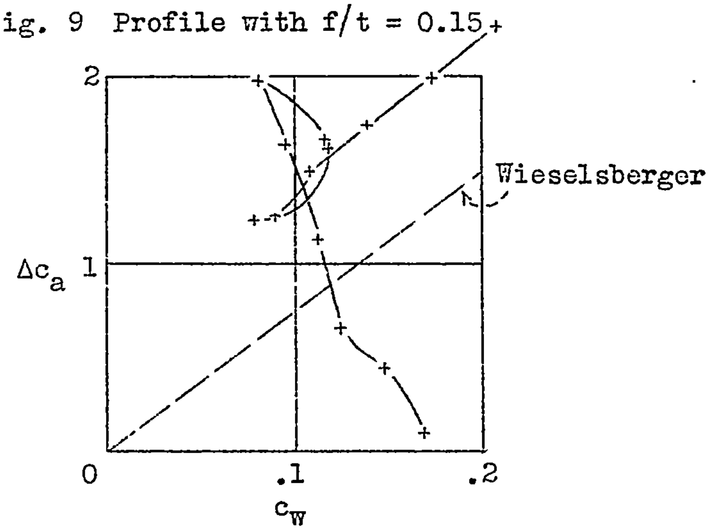

Fig. 10 Profile with f/t = 0.20 and d/t = 0.15

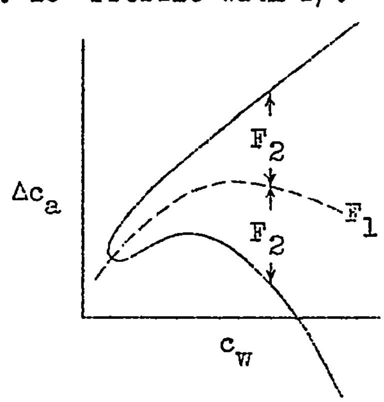

Fig. 11 Diagram showing relationship between Δc_a and c_w